

RESEARCH PAPER

Energy-based model of the Garofalo equation

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ABSTRACT

The determination of deformation resistance was conducted using a DIL805A/D quenching dilatometer equipped with a hydraulic unit, enabling compression tests during high-temperature forming at specified strain rates. Deformation resistance was experimentally defined for a 5x5 testing matrix, covering deformation temperatures (800, 900, 1000, 1100, and 1200 °C) and strain rates (0.001, 0.01, 0.1, 1, and 10 s⁻¹). Boron-alloyed steel, also known as BCT steel, was used as the experimental material. The Garofalo equation was employed to evaluate the experimental measurements of deformation resistance. This equation describes the hot metal forming process, where deformation resistance is a function of strain rate and temperature. The material constants of the Garofalo equation were determined based on peak stress values obtained from individual deformation curves. The technological process of metal forming is defined by three parameters: strain rate, deformation temperature, and strain level. The Garofalo equation does not include the last parameter, strain level, as an independent variable. Therefore, the Garofalo equation was modified into a strain-dependent model by transforming the material constants into functions of strain. Each material constant was expressed as an n-th degree strain polynomial. In this manner, the third independent variable, strain level, was introduced into the Garofalo equation. To improve the correlation between measured and calculated data, the degree of the strain polynomials was increased. Some publications have utilised strain polynomials up to the 14th degree; however, such a model requires up to 60 material constants. This paper presents the transformation of the Garofalo equation into an energy-based model. The core of this transformation lies in expressing the activation energy constant as a function of strain rate, temperature, and strain level. The material constants Alpha, n, and C remain unchanged, retaining their original meaning as defined by Garofalo. The activation energy is defined by 14 material constants. Together with the remaining three constants, a total of 17 material constants sufficiently define the incorporation of the strain level into the Garofalo equation. A 3D visualization of the energy model of the Garofalo equation showed that this equation has an extremum, a global maximum of deformation resistance.

Keywords: Garofalo equation; Boron steel; hot compression test; stress-strain curves; numerical simulations; activation energy

INTRODUCTION

The process of hot metal forming requires knowledge of the fundamental deformation resistance to calculate forming forces. The calculation of deformation force is based on technological parameters: forming temperature, strain rate, and strain level. Under laboratory conditions, physical modeling of the forming process for metallic materials is conducted using torsion or compression plastometers. The hot metal forming process was first described and published by Garofalo [1]. The equation takes the following form:

$$\dot{\varphi} \cdot \exp\left(\frac{Q}{RT}\right) = C [\sinh(\alpha\sigma_p)]^n \quad (1)$$

where: $\dot{\varphi}$ [s⁻¹] – strain rate
 T [K] – absolute temperature
 σ_p [MPa] – peak stress (flow stress)
 Q [J.mol⁻¹] – activation energy of deformation
 R [J.K⁻¹.mol⁻¹] – universal gas constant
 n [–] – material constant
 C [s⁻¹] – material constant
 α [MPa⁻¹] – material constant.

In this work, Garofalo focused more on the application of Eq. (1) to define the fundamentals of metal creep. A more detailed explanation of creep was later developed in his work [2]. The Garofalo equation was originally proposed for steady-state creep, where stress remains constant. However, during hot metal forming, the flow stress is also dependent on a third variable: the strain level. In the Garofalo (Eq. 1), the metal deformation process is described by two independent variables: strain rate and deformation temperature. The dependent variable is the flow stress, $\sigma = \sigma(\dot{\varphi}, T)$. During the hot forming process of metallic materials, hardening or softening of the formed material occurs. This material behavior is not described by the Garofalo equation, as it lacks the variable that would account for it – the strain level. This issue has proven to be a limitation of the Garofalo equation. Consequently, researchers have sought extensions of the Garofalo equation to include the strain level, or empirical functions of the type

$\sigma = \sigma(\varphi, T, \varphi)$, where φ represents strain. Numerous such empirical functions are documented in the literature [3–5]. A particularly detailed analysis of empirical functions describing deformation resistance is provided in [6].

Another option for describing a material's high-temperature deformation resistance is the Hensel-Spittel model [7]. It is formulated as a base value of deformation resistance, σ_0 , specific to a given material, multiplied by factors that account for the effects of strain rate, deformation temperature, and strain level. The resulting equation takes the form:

$$\sigma = \sigma_0 \cdot k_{\dot{\varphi}} \cdot k_T \cdot k_{\varphi} \quad (2)$$

where σ_0 represents the basic deformation resistance under standard conditions, when $\dot{\varphi} = 1 \text{ s}^{-1}$. The influence of strain rate on deformation resistance is expressed by the factor $k_{\dot{\varphi}}$, the influence of deformation temperature by the factor k_T , and the influence of strain level by the factor k_{φ} . These factors (coefficients) are defined as follows:

$$k_{\dot{\varphi}} = A_3 \dot{\varphi}^{m_3} \quad (3)$$

$$k_T = A_1 e^{-m_1 T} \quad (4)$$

$$k_{\varphi} = A_2 \varphi^{m_2} \quad (5)$$

where the constants A_1, A_2, A_3 and m_1, m_2, m_3 are material constants. In reference [7], these constants are listed for various metals and their alloys in the form of tables and graphs. It should be noted that at the time of the book's publication, the authors did not use the term "deformation resistance" but rather "deformation strength" (German: *Umformfestigkeit*). Since using tables and graphs was impractical, various empirical equations were introduced, such as:

$$\sigma = A \cdot t^{m_0} \cdot e^{m_1 T} \cdot \varphi^{m_2} \cdot e^{\frac{m_3}{\varphi}} \cdot (1 + \varphi)^{m_5} \cdot e^{m_7 \varphi} \cdot \dot{\varphi}^{(m_3 + m_8 t)} \quad (6)$$

where A and m_i through m_9 are material constants expressing the influence of forming conditions on flow stress. This is already an equation of the type $\sigma = \sigma(\dot{\phi}, T, \varphi)$, intended to address the deficiency of the Garofalo equation – the missing strain variable. A frequently utilized mathematical model for deformation resistance was introduced by Johnson and Cook [8]. The equation consists of the product of three components that influence deformation resistance. The first component $s(\dot{\phi})$ expresses the influence of strain rate, the second component $s(T)$ expresses the influence of deformation temperature, and the third component $s(\varphi)$ expresses the influence of strain level. The notation of these three components represents the equation:

$$\sigma = s(\dot{\phi}) \cdot s(T) \cdot s(\varphi) \quad (7)$$

The individual components were then defined as partial functions:

$$s(\dot{\phi}) = 1 + C \ln \dot{\phi}^* \quad (8)$$

$$s(T) = 1 - T^{*m} \quad (9)$$

$$s(\varphi) = A + B\varphi^n \quad (10)$$

where φ is the equivalent plastic strain, $\dot{\phi}^* = \dot{\phi}/\dot{\phi}_0$ is the dimensionless plastic strain rate for $\dot{\phi}_0 = 1.0 \text{ s}^{-1}$, and T^* is the homologous temperature. The five material constants are A, B, n, C, m . The expression in the first set of brackets gives stress as a function of strain for $\dot{\phi}_0 = 1.0$ and $T^* = 0$. The resulting equation for calculating deformation resistance then takes the form:

$$\sigma = (A + B\varphi^n)(1 + C \ln \dot{\phi}^*)(1 - T^{*m}) \quad (11)$$

This Eq. (11) does not contain a dimension carrier for deformation resistance, the unit MPa. Eq. (11) has undergone numerous modifications and adjustments [9–11]. Other solutions involved the reconstruction of the Garofalo equation. In professional literature, two research groups compete for priority in introducing the solution of replacing the material constants of the Garofalo equation with strain-dependent functions (strain compensation). One is the group led by Riccio [12]. All material constants, including the value of constant activation energy, were expressed as strain polynomials. The constants of the Garofalo equation were replaced by functions with the strain variable: $\alpha(\varphi)$, $n(\varphi)$, $Q(\varphi)$ and $C(\varphi)$. Thus, the strain-dependent model of the Garofalo equation was introduced. Eq. (1) was extended by the strain variable φ :

$$\dot{\phi} \cdot \exp\left(\frac{Q(\varphi)}{RT}\right) = C(\varphi)[\sinh(\alpha(\varphi) \cdot \sigma)]^{n(\varphi)} \quad (12)$$

The polynomials for the material constants were constructed for the AZ31 alloy [12] and are expressed in the following form:

$$1000\alpha(\varphi) = 26.414 - 48.421\varphi + 106.5\varphi^2 - 81.894\varphi^3 \quad (13)$$

$$n(\varphi) = 3.78 + 0.019\varphi - 9.235\varphi^2 + 8.639\varphi^3 \quad (14)$$

$$Q(\varphi) = 133.725 + 135.264\varphi - 458.078\varphi^2 + 243.788\varphi^3 \quad (15)$$

$$\ln C(\varphi) = 26.563 + 8.870\varphi - 58.980\varphi^2 + 37.568\varphi^3 \quad (16)$$

The second group is led by Lin [13]. They likewise confirmed the idea of implementing a strain-dependent model of the Garofalo equation. They increased the degree of the strain polynomials to 5. The mathematical form of these polynomials was as follows:

$$\alpha(\varphi) = A_0 + A_1\varphi + A_2\varphi^2 + A_3\varphi^3 + A_4\varphi^4 + A_5\varphi^5 \quad (17)$$

$$n(\varphi) = n_0 + n_1\varphi + n_2\varphi^2 + n_3\varphi^3 + n_4\varphi^4 + n_5\varphi^5 \quad (18)$$

$$Q(\varphi) = Q_0 + Q_1\varphi + Q_2\varphi^2 + Q_3\varphi^3 + Q_4\varphi^4 + Q_5\varphi^5 \quad (19)$$

$$\ln C(\varphi) = C_0 + C_1\varphi + C_2\varphi^2 + C_3\varphi^3 + C_4\varphi^4 + C_5\varphi^5 \quad (20)$$

Specific numerical values for the individual constants used in Eq. (17) through Eq. (20) were established for 42CrMo steel and are listed in Table 1. This strain-dependent Garofalo model required the introduction of up to 24 material constants into the Garofalo equation. Although the strain-dependent model successfully incorporated strain into the equation, subsequent investigations and calculations revealed that 3rd and 5th-degree strain polynomials did not resolve the issue of reliable prediction for deformation resistance. Third-degree strain polynomials were published in [12, 14]. For H62 brass, a 4th-degree strain polynomial was applied [11]. The most frequently used degree for strain polynomials is 5 [15–19]. Further publications utilized 6th-degree strain polynomials [20–22]. The

refinement of strain polynomials continued, with reference [23] recording a degree of $n=9$. In publications [24, 25], the degree of the strain polynomials reached $n=14$. This degree of strain polynomials introduces nearly 60 material constants into the Garofalo equation. Recently, neural networks have been employed to refine the constants of strain polynomials [26, 27].

Table 1 Material constants of 42CrMo steel for Eq. (17) to Eq. (20) [13].

i	Q_i	n_i	C_i	Alfa_i
0	319.897	8.470	28.853	0.011
1	1943.855	12.134	173.875	-0.033
2	-9790.029	-111.366	-870.200	0.136
3	22036.812	288.650	1953.277	-0.260
4	-23042.790	-313.310	-2041.080	0.246
5	9055.826	125.400	802.221	-0.093

In [28], the authors proposed a mathematical model as a function of stress depending on strain:

$$\sigma = \sigma_0 \varphi^{a_0} e^{F(\varphi)} \Big|_{\varphi, t} \quad (21)$$

where: σ [MPa] – flow stress (deformation resistance)
 σ_0 [MPa] – basic deformation resistance
 a_0 [–] – material constant
 $F(\varphi)$ [–] – suitably chosen deformation function
 φ [–] – logarithmic deformation.

The function $F(\varphi)$ used in Eq. (21) allows for the definition of the mathematical model as an open model that can be adapted to describe measured data. The solution was found in the use of the Laurent polynomial

$$F(\varphi) = b_{III}\varphi^{-3} + b_{II}\varphi^{-2} + b_I\varphi^{-1} + b_0 + b_1\varphi + b_2\varphi^2 + b_3\varphi^3 \quad (22)$$

where b_{III} , b_{II} , b_I , b_0 , b_1 , b_2 and b_3 are dimensionless material constants. In further research, the authors [18] expanded Eq. (21) to include two independent variables: deformation resistance as a function of strain and deformation temperature at a constant strain rate. Eq. (21) was extended by the deformation temperature variable t . The constant a_0 was replaced by a suitably chosen temperature function $E(t)$, and the F -function was supplemented with the deformation temperature. Eq. (21) then took a new form:

$$\sigma = \sigma_0 \varphi^{E(t)} e^{F(\varphi, t)} \Big|_{\varphi} \quad (23)$$

where: σ [MPa] – flow stress (deformation resistance)
 σ_0 [MPa] – basic deformation resistance
 t [°C] – deformation temperature
 $E(t)$ [–] – suitably chosen function of the deformation temperature,
 $F(\varphi, t)$ [–] – suitably chosen function of deformation and temperature.

The following equation proved to be a suitable E -function:

$$E(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \quad (24)$$

The dimensions of constants a_0 through a_3 are chosen to compensate for the temperature dimension to a dimensionless value. In some cases, it proved appropriate to transform the deformation temperature t into a transformed value of $\ln t$ or $t/1000$. The F -function was supplemented with the deformation temperature variable:

$$F(\varphi, t) = b_1\varphi^{-3} + b_2\varphi^{-2} + b_3\varphi^{-1} + b_4\varphi + b_5\varphi^2 + b_6\varphi^3 + b_7 t + b_8 t^2 + b_9 t^3 + (b_{10}\varphi^{-3} + b_{11}\varphi^{-2} + b_{12}\varphi^{-1} + b_{13}\varphi + b_{14}\varphi^2 + b_{15}\varphi^3)t^{-1} \quad (25)$$

The dimensions of constants b_1 through b_{15} are chosen to compensate for the temperature dimension to a dimensionless value. Equation (23) was constructed for deformation resistance $\sigma = \sigma(\varphi, t)|_{\varphi}$. In this equation, we substitute deformation temperature with strain rate, resulting in the equation $\sigma = \sigma(\dot{\phi}, \varphi)|_t$ [28]:

$$\sigma = \sigma_0 \varphi^{E(\dot{\phi})} e^{F(\varphi, \dot{\phi})} \quad (26)$$

where: σ [MPa] – flow stress (deformation resistance)
 σ_0 [MPa] – basic deformation resistance

$\dot{\phi}$ [s⁻¹] – strain rate

$E(\dot{\phi})$ [–] – suitably chosen strain rate function

$F(\phi, \dot{\phi})$ [–] – suitably chosen deformation and deformation rate function

The following equation proved to be a suitable E -function:

$$E(\dot{\phi}) = a_0 + a_1\dot{\phi} + a_2\dot{\phi}^2 + a_3\dot{\phi}^3 \quad (27)$$

The dimensions of constants a_0 through a_3 are chosen to compensate for the strain rate dimension to a dimensionless value. The F -function is supplemented with the variable t – deformation temperature:

$$F(\phi, \dot{\phi}) = b_1\phi^{-2} + b_2\phi^{-1} + b_3\phi + b_4\phi^2 + b_5\phi^3 + b_6\dot{\phi} + b_7\dot{\phi}^2 + b_8\dot{\phi}^3 + (b_9\phi + b_{10}\dot{\phi}\phi^2 + b_{11}\phi^{-1} + b_{12}\phi^{-2} + b_{13}\phi^{-3})\dot{\phi} \quad (28)$$

The dimensions of constants b_1 through b_{13} are chosen to compensate for the strain rate and strain level dimensions to a dimensionless value. Eq. (23) and Eq. (26) achieved relatively good results, but the high number of coefficients was demanding for mathematical calculation. One advantage of this mathematical model is that it does not require non-linear regression to calculate the coefficients; linear regression is sufficient. Trial calculations were performed with the function $\sigma = \sigma(\dot{\phi}, t, \phi)$ [28]:

$$\sigma = \sigma_0 \phi^{E(\dot{\phi}, t, \phi)} e^{F(\phi, \dot{\phi}, t, \phi)} \quad (29)$$

These trial calculations required a high number of constants and still did not yield satisfactory results. For BCT steel, material constants for the strain-dependent Garofalo model were published in [29]. The coefficients of the strain polynomials for Eq. (17) through Eq. (20) are listed in Table 2.

Table 2 Material constants of BCT steel for Eq. (17) to Eq. (20) [29].

i	Alfa _i (MPa ⁻¹)	n _i (–)	Q _i (kJ.mol ⁻¹)	C _i (ln s ⁻¹)
0	0.009723	7.315676	437.738387	39.160611
1	0.001981	-22.311054	-1590.550978	-160.25097
2	-0.049054	72.344513	5949.512220	13.771766
3	0.172226	-116.374882	-11467.052987	-1192.299514
4	-0.214534	95.405015	11229.646740	1167.051242
5	0.089835	-31.880095	-4375.469961	-452.658465
R ²	0.9946	0.9984	0.9977	0.9985

EXPERIMENTAL MATERIAL AND METHODS

Boron-alloyed steel, working-titled BCT steel, was used as experimental material. The chemical composition of the steel is presented in Table 3. Compression tests for measuring deformation resistance were conducted using a DIL805A/D dilatometer. Geometrically, the test specimen consisted of a cylinder with a diameter of $D = 5$ mm, and a length of $L = 10$ mm. Specimen heating was provided by an electrical induction system with a defined heating rate. Deformation curves were measured at temperatures of 800, 900, 1000, 1100, and 1200 °C. The logarithmic strain was continuously measured within the range of 0 to 0.9. Specific

strain rates of 0.001, 0.01, 0.1, 1 and 10 s⁻¹ were selected for the deformation resistance measurements. In terms of deformation temperature and strain rate, a full 5x5 experimental matrix was employed. For the numerical evaluation of the compression test process, the Garofalo equation was utilized, modified into an energy-based model. The essence of transitioning to this new energy-based Garofalo model lies in introducing activation energy as a functional dependence on strain rate, deformation temperature, and strain level. The implementation of this new method for defining activation energy enabled a realistic prediction of

Table 3 Chemical composition of BCT steel (mass %).

Element	C	Mn	Si	P	Cr	V	Mo
Composition	0.12	1.60	0.45	0.013	0.04	0.002	0.02
Element	Ni	Nb	Al	S	W	B	Fe
Composition	0.07	0.04	0.03	0.005	0.01	0.002	balance

deformation curves. The proposed mathematical model of the Garofalo equation was constructed such that the calculation of material constants does not require the use of non-linear regression.

RESULTS AND DISCUSSION

As previously noted, the Garofalo equation has a certain limitation regarding plasticity, as it does not include a variable expressing the strain value ϕ . Previous studies sought a solution by introducing material constants into the Garofalo equation as functions of strain: $\alpha(\phi)$, $n(\phi)$, $C(\phi)$ and $Q(\phi)$. These strain-dependent functional constants modified the basic Garofalo Eq. (1), resulting in the strain-dependent Garofalo model Eq. (12). This work, however, keeps the material constants α , n and C unchanged, performing their functions as in the basic Garofalo model Eq. (1). The central idea of this solution is an analogy with Eq. (2) and Eq. (7), where the activation energy term in the Garofalo equation is composed of the product of three components:

$$\frac{Q}{RT} = e^{\frac{Q(\phi)}{RT}} \cdot e^{\frac{Q(T)}{RT}} \cdot e^{\frac{Q(\phi)}{RT}} \quad (30)$$

where the component $Q(\phi)$ represents the portion of activation energy attributed to the effect of strain rate, $Q(T)$ represents the portion attributed to the effect of deformation temperature, and $Q(\phi)$ represents the portion attributed to the effect of strain. The material constant, activation energy, is introduced into the Garofalo equation as a function $Q = Q(\phi, T, \phi)$. Mathematically this is expressed as:

$$Q = Q(\phi, T, \phi) = Q(\phi) + Q(T) + Q(\phi) \quad (31)$$

The fact that each activation energy component is formed by a separate function proved to be an advantage for mathematical calculations in further stages. Previous researchers utilized polynomials for strain. For the activation energy component dependent on strain rate, a 3rd-degree polynomial was proposed. Instead of using the strain rate directly, a substitution in the form of the natural logarithm of the strain rate was applied:

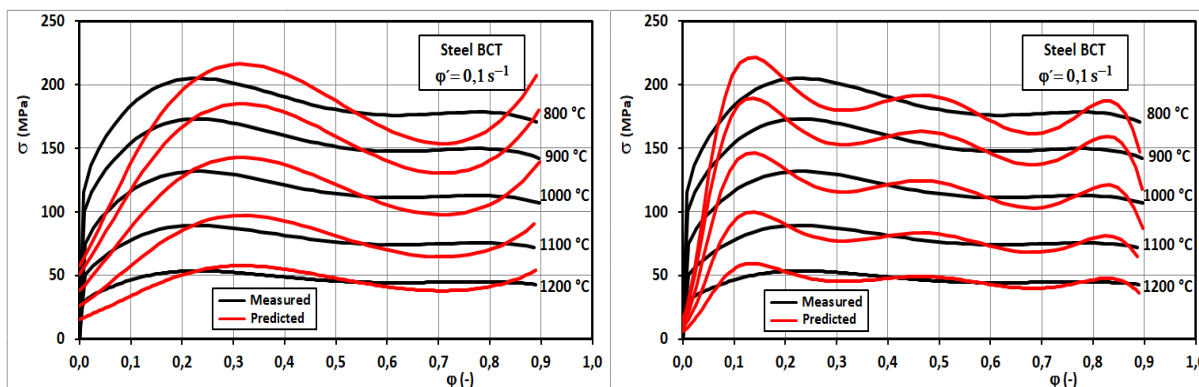


Fig. 1 Effect of activation energy from deformation on predicted curves, (a) $Q(\phi)$ – 3rd degree polynomial, (b) $Q(\phi)$ – 6th degree polynomial.

$$Q(\dot{\phi}) = c_0 + c_1 \ln \dot{\phi} + c_2 \ln^2 \dot{\phi} + c_3 \ln^3 \dot{\phi} \quad (32)$$

where c_0 , c_1 , c_2 and c_3 are material constants. For the activation energy component dependent on deformation temperature, a 2nd-degree polynomial was used, where the forming temperature must be given in [K] rather than [°C]:

$$Q(T) = a_0 + a_1 T + a_2 T^2 \quad (33)$$

where a_0 , a_1 and a_2 are material constants. This relationship is based on the work [29], where it was demonstrated—using measured data from compression tests—that activation energy depends on temperature according to a parabolic law.

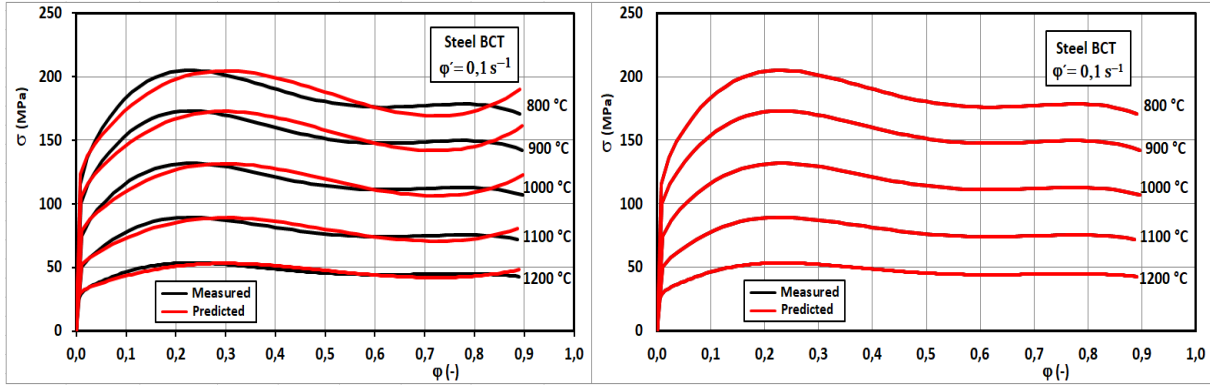


Fig. 2 Deformation curves, Laurent series, (a) level 3, (b) level 4.

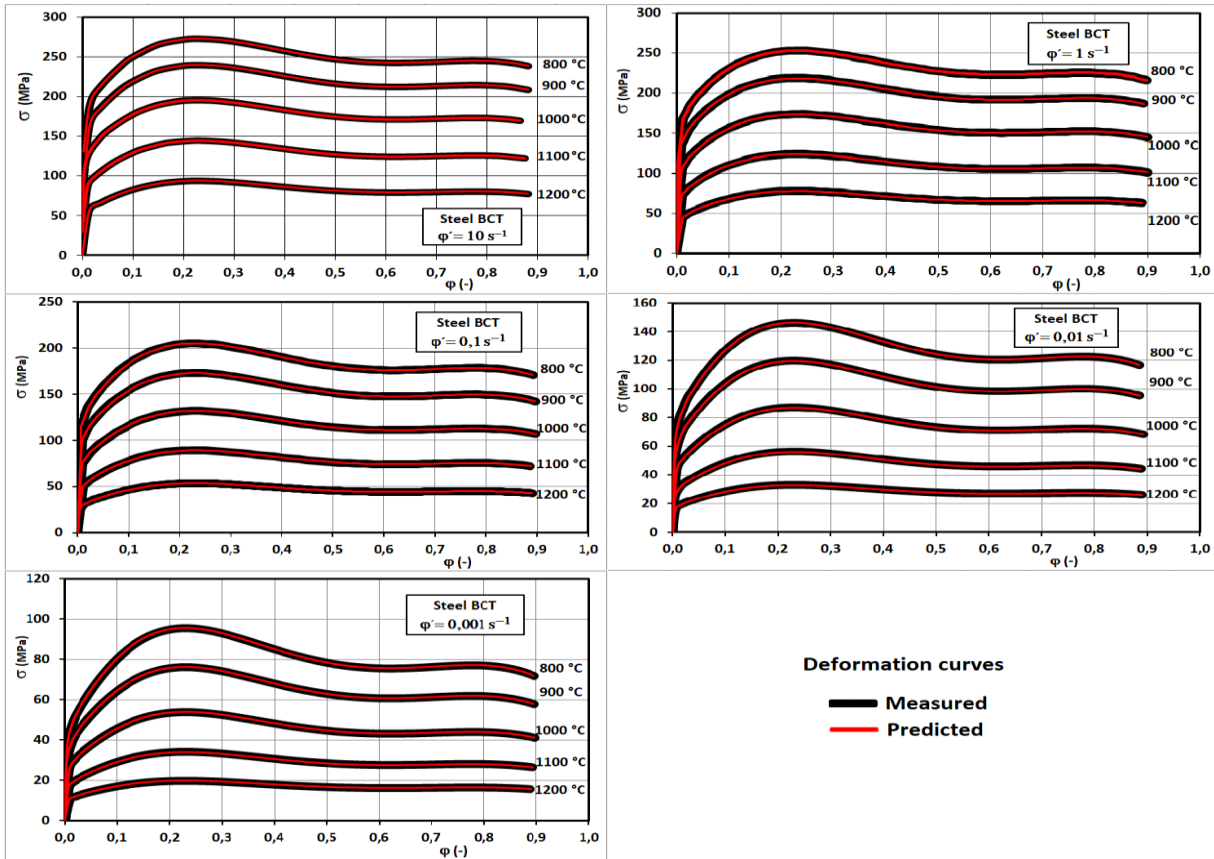


Fig. 3 Deformation curves of BCT steel.

The conclusions of work [29] became the initiating idea: that activation energy cannot be viewed as a point value (constant), but rather as a functional value of the technological parameters of the deformation process (temperature, strain rate, and strain level). Establishing the functional dependence of the activation energy component on the strain value initially appeared to be an almost insurmountable problem. The initially applied 3rd-degree polynomial failed, as documented by the graph in Fig. 1a. Increasing the polynomial to the 6th degree also failed (see Fig. 1b).

It was confirmed that polynomials of degrees higher than 3 become unstable, as they do not accurately reflect the function's behavior between node points. An unconventional solution was adopted: using a Laurent series instead of polynomials. For calculating the strain-dependent activation energy component, a 3rd-degree Laurent series was deployed, resulting in substantial improvement. The final solution was found in the use of a 4th-degree Laurent series:

$$Q(\phi) = b_{IV}\phi^{-4} + b_{III}\phi^{-3} + b_{II}\phi^{-2} + b_I\phi^{-1} + b_0 + b_1\phi + b_2\phi^2 + b_3\phi^3 + b_4\phi^4 \quad (34)$$

where b_{IV} , b_{III} , b_{II} , b_I , b_0 , b_1 , b_2 , b_3 and b_4 are material constants. The complete form of the equation for calculating activation energy is as follows:

$$Q = Q(\phi, T, \varphi) = a_0 + a_1 T + a_2 T^2 + c_1 \ln \phi + c_2 \ln^2 \phi + c_3 \ln^3 \phi + b_{IV} \varphi^{-4} + b_{III} \varphi^{-3} + b_{II} \varphi^{-2} + b_I \varphi^{-1} + b_1 \varphi + b_2 \varphi^2 + b_3 \varphi^3 + b_4 \varphi^4 \quad (35)$$

The basic Garofalo Eq. (1) is thus transformed into the **energy-based Garofalo model** in the form:

$$\dot{\phi} \cdot \exp\left(\frac{Q(\phi, T, \varphi)}{RT}\right) = C[\sinh(\alpha\sigma)]^n \quad (36)$$

To calculate the material constants, it is suitable to linearize Eq. (36) using natural logarithms:

$$\ln \dot{\phi} + \frac{1}{RT}(Q(\phi) + Q(T) + Q(\varphi)) = \ln C + n \ln(\sinh(\alpha\sigma)) \quad (37)$$

This form of the Garofalo equation creates favorable conditions for calculating material constants. From a mathematical perspective, multiple linear regression is utilized for this purpose. Eq. (37) allows for the calculation of material constants without the use of non-linear regression or neural networks. The material constant α is determined from the peak stress. The detailed procedure for calculating the constant α was published in [30–32]. For BCT steel, the material constant α has a value of $\alpha=0.009034 \text{ MPa}^{-1}$. Additional material constants for Eq. (36) are listed in **Table 4**. A graphical comparison of measured and calculated deformation resistance values for a strain rate of 0.1 s^{-1} is shown in **Fig. 2**. For the construction of **Fig. 2a**, the activation energy component $Q(\phi)$ defined by a 3rd-degree Laurent polynomial was used. For **Fig. 2b**, a 4th-degree Laurent polynomial was used. The approximation of measurement results using the Laurent polynomial showed a convergence of the curves from measured and calculated values.

Table 4 Material constants of BCT steel for Eq. (36).

Constant	Unit	Laurent polynomial of degree 3	Laurent polynomial of degree 4
α	MPa^{-1}	0.009034161	0.009034161
n	–	7.534922260	6.821044214
C	s^{-1}	1.278891E+11	1.278891E+11
a_0	J.mol^{-1}	-614324.3251	-552256.2417
a_1	$\text{J.mol}^{-1}.\text{T}^{-1}$	1534.677312	1395.976246
a_2	$\text{J.mol}^{-1}.\text{T}^{-2}$	-0.649329788	-0.581366363
c_1	$\text{J.mol}^{-1}.\ln \text{ s}^{-1}$	814.4209857	-5.414893E-06
c_2	$\text{J.mol}^{-1}.\ln^2 \text{ s}^{-1}$	-1667.212743	-1506.314204
c_3	$\text{J.mol}^{-1}.\ln^3 \text{ s}^{-1}$	-113.969762	-108.274771
b_{IV}	J.mol^{-1}	–	3.200395E-13
b_{III}	J.mol^{-1}	-1.17556E-08	-7.196385E-08
b_{II}	J.mol^{-1}	2.102989E-03	4.472926E-03
b_I	J.mol^{-1}	-71.37437744	-90.64224951
b_1	J.mol^{-1}	386509.9388	533069.7565
b_2	J.mol^{-1}	-919550.6794	-1940363.6636
b_3	J.mol^{-1}	601812.8250	2624151.3097
b_4	J.mol^{-1}	–	-1210146.6853
R^2	–	0.994 378	0.999 999

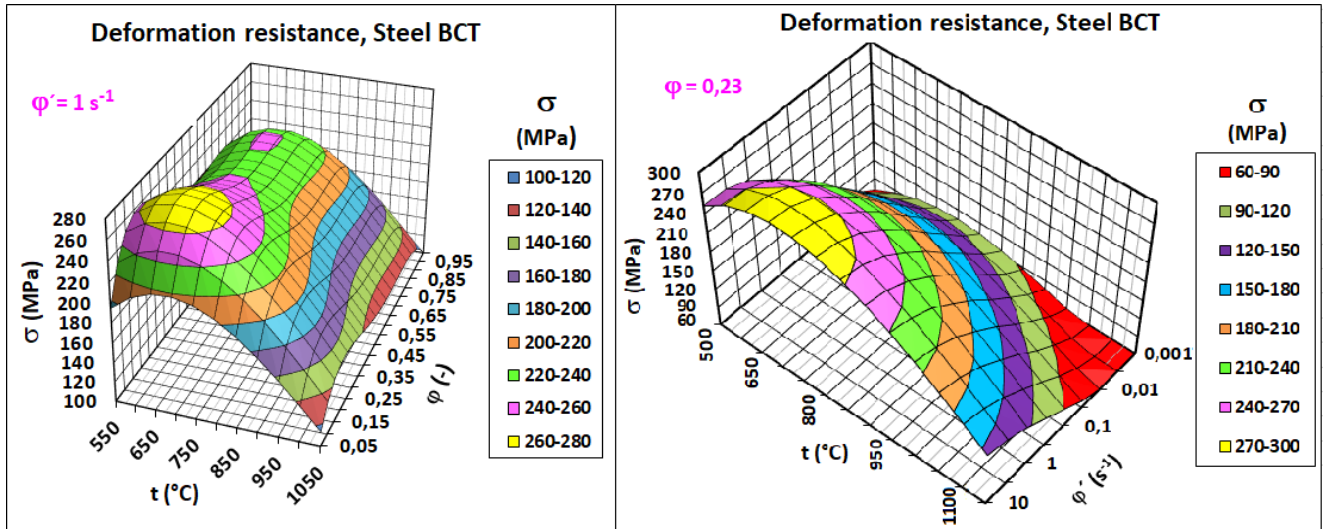


Fig. 4 Deformation resistance, 3D visualization of BCT steel, (a) coordinates $\sigma=\sigma(T, \varphi)$, (b) coordinates $\sigma=\sigma(T, \varphi')$.

The extension of the curves displaying the measured data demonstrated that Eq. (36) is a functional dependence of deformation resistance in high-temperature metal forming. A 2D visualization of deformation curves for all five strain rates (10, 1, 0.1, 0.01 and 0.001 s^{-1}) is provided in **Fig. 3**. The correlation coefficient of these curves approaches a value of $R^2 \approx 1$. This value confirmed that introducing strain into the Garofalo equation via activation energy is a suitable solution. The graph in **Fig. 4** shows a 3D visualization of the deformation resistance of BCT steel. **Fig. 4a** represents the visualization for the function $\sigma = \sigma(T, \varphi)|_{\varphi}$, where the strain rate is constant at 1 s^{-1} . **Fig. 4b** represents the visualization for the function $\sigma = \sigma(T, \varphi)|_{\varphi}$, where the strain is constant at $\varphi=0.23$. **Fig. 4a** documents that the deformation resistance defined according to Eq. (36) has an extremum in the form of a maximum. At certain coordinates (strain rate, temperature, and strain level), it reaches a maximum of deformation resistance.

The exact position of the maximum deformation resistance coordinates cannot be determined directly from the graph in **Fig. 4**. Therefore, it is necessary to perform a graphical analysis of equation (36). We will sequentially perform 2D visualizations of the deformation resistance curves from Eq. (36) under certain conditions of two constant variables. **Fig. 5a** shows the deformation resistance function $\sigma = \sigma(\varphi)|_{\varphi, T}$, confirming the existence of a maximum deformation resistance depending on the strain rate. Similarly, **Fig. 5b** shows the deformation resistance function $\sigma = \sigma(T)|_{\varphi, \varphi'}$, confirming the existence of a maximum deformation resistance depending on the deformation temperature. **Fig. 6a** displays the deformation resistance function as a function of strain, $\sigma = \sigma(\varphi)|_{T, \varphi'}$, which indicates the existence of a deformation resistance maximum. The difference between the strain-dependent Garofalo model and the energy-based Garofalo model is illustrated by the 3D visualization of Eq. (12) with polynomials Eq. (17) to Eq. (20), as shown in **Fig. 7**.

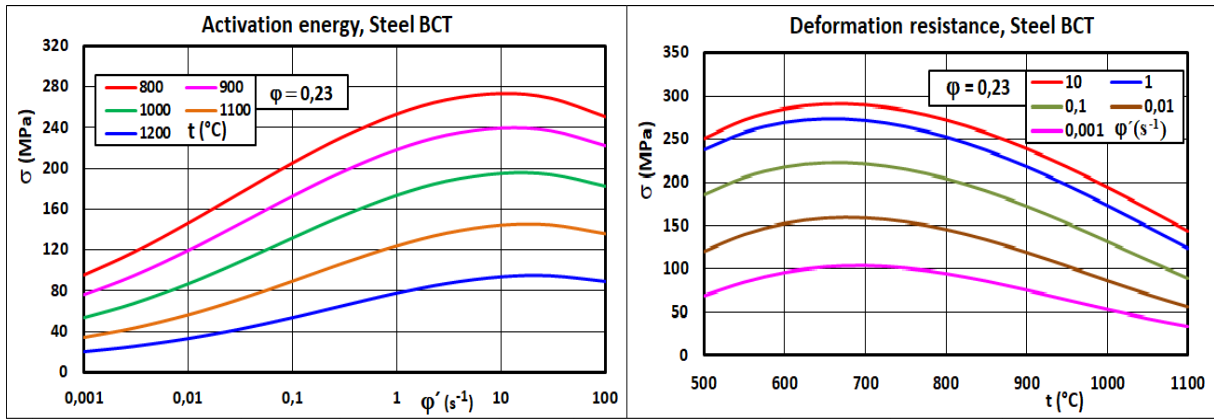


Fig. 5 Curves passing through the maximum deformation resistance, (a) deformation rate $\sigma = \sigma(\dot{\phi}')$, (b) $\sigma = \sigma(T)$ deformation temperature

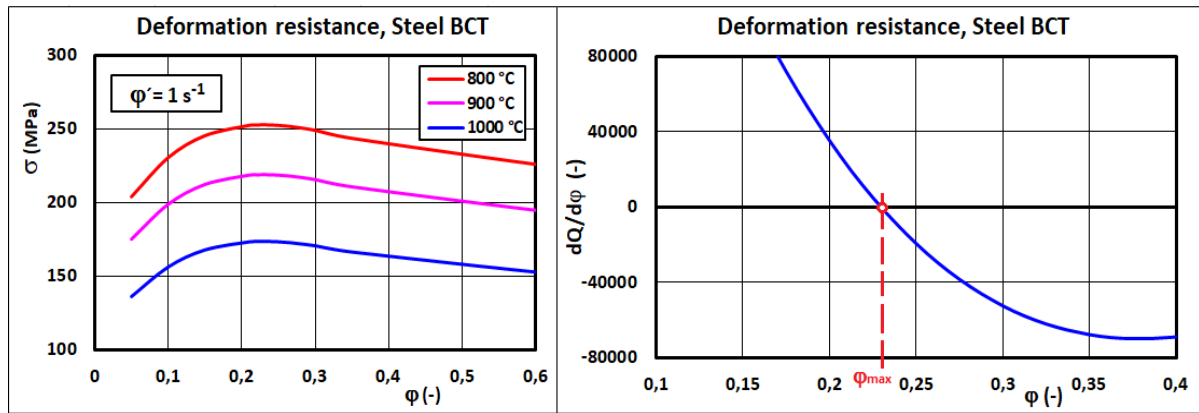


Fig. 6 Effect of deformation on the position of the maximum deformation resistance, (a) deformation $\sigma = \sigma(\phi)$, (b) deformation coordinate $\phi_{\max}$

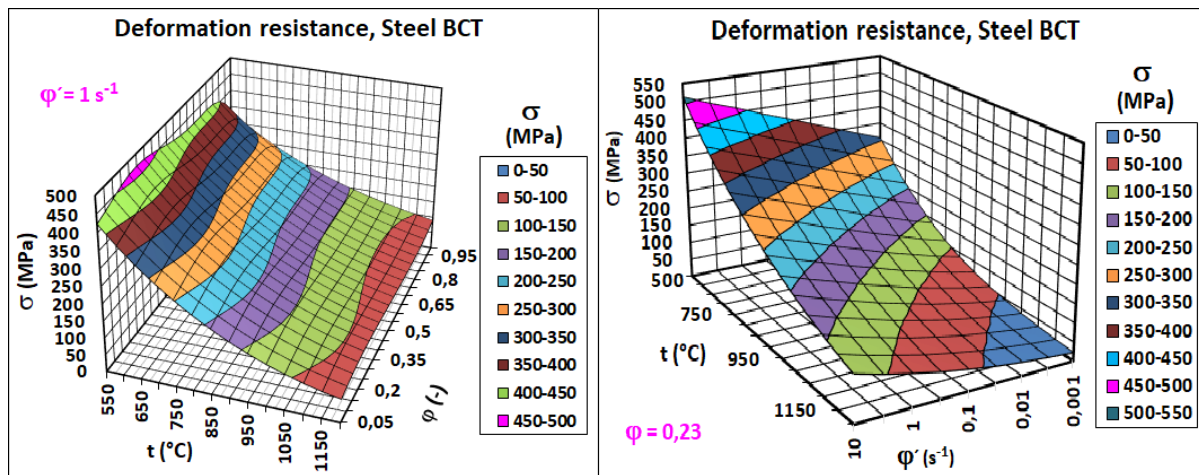


Fig. 7 Visualization of the Garofalo equation deformation model for BCT steel.

The specific constants are provided in **Table 2**. The mathematical model Eq. (12) forms a smooth surface without a deformation resistance extremum. To calculate the specific numerical values of the coordinates for the deformation resistance maximum (strain rate, deformation temperature, and strain level), an analytical analysis of Eq. (36) was performed. For function Eq. (36) to possess an extremum, the following must hold for all three coordinates:

$$\frac{\partial \sigma(\dot{\varphi}, T, \varphi)}{\partial \dot{\varphi}} = 0, \quad \frac{\partial \sigma(\dot{\varphi}, T, \varphi)}{\partial T} = 0, \quad \frac{\partial \sigma(\dot{\varphi}, T, \varphi)}{\partial \varphi} = 0 \quad (38)$$

The calculation involves partial differentiation of Eq. (36). Due to the extensive mathematical derivation, only the resulting relationships are presented. In the first step, we determine the strain coordinate φ_{max} for the deformation resistance maximum:

$$-4b_{IV}\varphi^{-5} - 3b_{III}\varphi^{-4} - 2b_{II}\varphi^{-3} - b_I\varphi^{-2} + b_1 + 2b_2\varphi + 3b_3\varphi^2 + 4b_4\varphi^3 = 0 \quad (39)$$

Eq. (39) represents an 8th-degree polynomial. Its analytical solution is unfeasible. Therefore, a graphical or numerical solution must be considered. An example of the graphical solution is shown in **Fig. 6b**. The strain coordinates defining the maximum deformation resistance of Eq. (36) has a value of $\varphi_{max} = 0.2298$. Deformation temperature defining the maximum deformation resistance is determined from the relationship:

$$T_{max} = \sqrt{\frac{1}{a_2}(a_0 + Q(\dot{\varphi}_{max}) + Q(\varphi_{max}))} \quad (40)$$

For a real extremum to exist, the following condition must be met: $(a_0 + Q(\dot{\varphi}_{max}) + Q(\varphi_{max}))/a_2 \geq 0$. The strain-dependent activation energy component $Q(\varphi_{max})$ can already be calculated since φ_{max} is known.

However, the strain rate-dependent activation energy component $Q(\dot{\varphi}_{max})$ is not yet known; therefore, the strain rate coordinate $\dot{\varphi}_{max}$ defining the deformation resistance maximum must be established:

$$\dot{\varphi}_{max} = \exp\left(\frac{1}{c_3}\left(-c_2 \pm \sqrt{c_2^2 - 3c_3(c_1 + RT_{max})}\right)\right) \quad (41)$$

A real extremum exists if the condition $c_2^2 \geq 3c_3(c_1 + RT_{max})$ is satisfied. Eq. (40) and Eq. (41) for calculating the deformation temperature and strain rate coordinates are coupled in a non-linear relationship. To obtain specific numerical values, these two equations must be solved using iterative numerical methods. The maximum deformation resistance value is calculated from the coordinates $\sigma_{max} = \sigma(\dot{\varphi}_{max}, T_{max}, \varphi_{max})$ using the following equation:

$$\sigma_{max} = \frac{1}{\alpha} \operatorname{argsinh}\left(\left(\frac{\dot{\varphi}_{max}}{c} \exp\left(\frac{Q(\dot{\varphi}_{max}, T_{max}, \varphi_{max})}{RT_{max}}\right)\right)^{1/n}\right) \quad (42)$$

Numerical calculations derived from the proposed Garofalo model (36) revealed that the maximum normal stress $\sigma_{max} = 294.74$ MPa was reached under the conditions $\dot{\varphi}_{max} = 8.245 \text{ s}^{-1}$ and $t_{max} = 665.18 \text{ }^\circ\text{C}$. Given that the experimental measurements were conducted within the plastic deformation condition intervals [$t \in <800; 1200> \text{ }^\circ\text{C}$, $\dot{\varphi} \in <0; 0.9>$, $\varphi \in <1.10^{-3}; 10> \text{ s}^{-1}$], and the calculated extremum value $\sigma_{max} = 294.74$ MPa was reached at a temperature $t_{max} = 665.18 \text{ }^\circ\text{C}$ —which lies outside the investigated interval—the calculated extremum should be considered a probable value. A comparison of measured and calculated deformation resistance values across the entire range of temperatures (800 to 1200 $^\circ\text{C}$), strain rates (0.001 to 10 s^{-1}), and strains (0 to 0.9) is presented in **Fig. 8**.

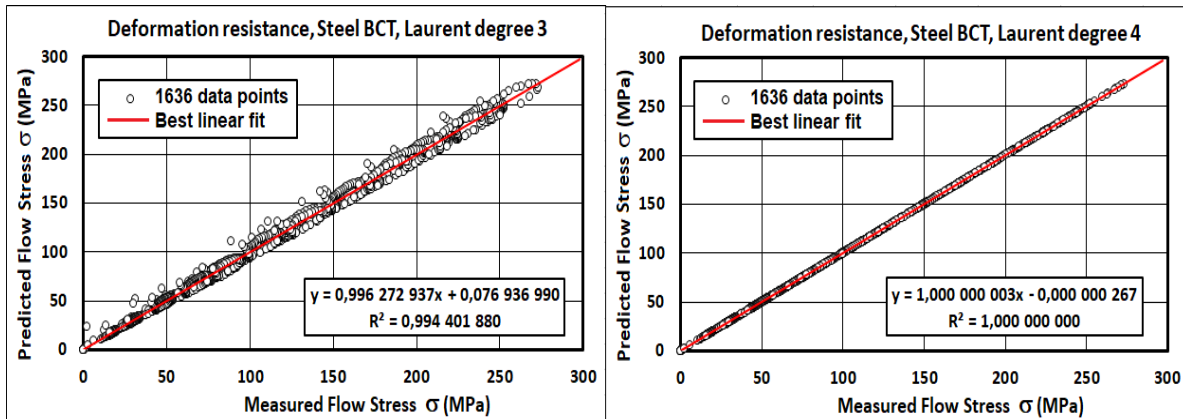


Fig. 8 Correlation between experimental data and the energy model of the Garofalo equation – Laurent series, (a) degree 3, (b) degree 4.

CONCLUSIONS

Physical modeling of the metal deformation process was conducted using compression tests for BCT steel. The testing scope consisted of a full 5x5 matrix, with measurements performed at five different deformation temperatures (800, 900, 1000, 1100, and 1200 $^\circ\text{C}$) and five different strain rates (0.001, 0.01, 0.1, 1, and 10 s^{-1}). A total of 25 deformation curves were obtained. The Garofalo equation was employed to evaluate the measured deformation resistance data, as it effectively describes the hot metal forming process. From a technological standpoint, however, the Garofalo equation has a known limitation: it does not include the strain-level variable. Previous solutions replaced the material constants in Eq. (1) with functional dependencies using strain polynomials. To increase the accuracy of deformation resistance calculations, the degree of these polynomials gradually increased. Publications [24, 25] report the use of strain polynomials up to $n=14$, which requires determining approximately 60 constants for the Garofalo equation. This work established the following criteria for calculating material constants: the calculation must be performed without non-linear regression, the number of constants must be minimized (without reducing calculation accuracy), and the constants should not require refinement via neural networks.

The proposed solution replaces the strain-dependent Garofalo model with a new energy-based Garofalo model. The core of this approach lies in reverting the material constants α , C , and n from functional strain polynomials back to constants. Instead, activation energy was introduced as a functional dependence on strain rate, deformation temperature, and strain level. Thus, the Garofalo equation took a new form, as defined by Eq. (36). Graphical and mathematical analyses of this equation demonstrated that, under certain conditions, deformation resistance can attain a global maximum. The energy-based Garofalo model achieves high accuracy in predicting deformation resistance using only 17 constants.

The utilization of the newly proposed mathematical model and the calculation of material constants enabled the transformation of the strain-dependent Garofalo model into an energy-based Garofalo model.

The new definition of activation energy as a function of strain rate, deformation temperature, and strain level enabled the calculation of activation energy under specific forming process conditions.

The energy-based Garofalo model will enable more precise numerical calculations in Finite Element Method (FEM) simulations.

Analysis of the energy-based Garofalo model revealed a global maximum in deformation resistance.

The newly proposed energy-based model significantly reduced the number of material constants while maintaining the reliability of deformation-resistance predictions.

Future research will focus on analyzing the dependence of plastic deformation activation energy on strain rate, temperature, and strain level, seeking applications that pair new insights into activation energy with deformation resistance.

Furthermore, it would be beneficial to explore how to link the energy-based Garofalo model to the chemical composition of the formed steels.

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